Instructional Objectives:

At the end of this lesson, the students should be able to understand:

- Definition of shaft
- Standard shaft sizes
- Standard shaft materials
- Design of shaft based on strength

8.1.1 Shaft

Shaft is a common and important machine element. It is a rotating member, in general,
has a circular cross-section and is used to transmit power. The shaft may be hollow or solid. The shaft is supported on bearings and it rotates a set of gears or pulleys for the purpose of power transmission. The shaft is generally acted upon by bending moment, torsion and axial force. Design of shaft primarily involves in determining stresses at critical point in the shaft that is arising due to aforementioned loading. Other two similar forms of a shaft are axle and spindle.

Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque. Spindle is simply defined as a short shaft. However, design method remains the same for axle and spindle as that for a shaft.

8.1.2 Standard sizes of Shafts

Typical sizes of solid shaft that are available in the market are,

Up to 25 mm	0.5 mm increments
25 to 50 mm	1.0 mm increments
50 to 100 mm	2.0 mm increments
100 to 200 mm	5.0 mm increments

8.1.3 Material for Shafts

The ferrous, non-ferrous materials and non metals are used as shaft material depending on the application. Some of the common ferrous materials used for shaft are discussed below.

Hot-rolled plain carbon steel

These materials are least expensive. Since it is hot rolled, scaling is always present on the surface and machining is required to make the surface smooth.

Cold-drawn plain carbon/alloy composition

Since it is cold drawn it has got its inherent characteristics of smooth bright finish. Amount of machining therefore is minimal. Better yield strength is also obtained. This is widely used for general purpose transmission shaft.

Alloy steels

Alloy steel as one can understand is a mixture of various elements with the parent steel to improve certain physical properties. To retain the total advantage of alloying materials one requires heat treatment of the machine components after it has been manufactured. Nickel, chromium and vanadium are some of the common alloying materials. However, alloy steel is expensive.

These materials are used for relatively severe service conditions. When the situation demands great strength then alloy steels are used. They have fewer tendencies to crack, warp or distort in heat treatment. Residual stresses are also less compared to CS(Carbon Steel).

In certain cases the shaft needs to be wear resistant, and then more attention has to be paid to make the surface of the shaft to be wear resistant. The common types of surface hardening methods are,

Hardening of surface

Case hardening and carburizing
Cyaniding and nitriding

8.1.4 Design considerations for shaft

For the design of shaft following two methods are adopted,

Design based on Strength

In this method, design is carried out so that stress at any location of the shaft should not exceed the material yield stress. However, no consideration for shaft deflection and shaft twist is included.

Design based on Stiffness

Basic idea of design in such case depends on the allowable deflection and twist of the shaft.

8.1.5 Design based on Strength

The stress at any point on the shaft depends on the nature of load acting on it. The stresses which may be present are as follows.

Basic stress equations :

Bending stress

$$\sigma_b = \frac{32M}{\pi d_0^3 (1 - k^4)}$$

(8.1.1)

Where,

M : Bending moment at the point of interest

d_o : Outer diameter of the shaft

k : Ratio of inner to outer diameters of the shaft (k = 0 for a solid shaft because inner diameter is zero)

Axial Stress

$$\sigma_a = \frac{4\alpha F}{\pi d_0^2 (1 - k^2)}$$

(8.1.2)

Where,

F: Axial force (tensile or compressive)

α : Column-action factor(= 1.0 for tensile load)

The term α has been introduced in the equation. This is known as column action factor. What is a column action factor? This arises due the phenomenon of buckling of long slender members which are acted upon by axial compressive loads.

Here, α is defined as,

$$\alpha = \frac{1}{1 - 0.0044(L/K)} \quad \text{for } L/K < 115$$

$$\alpha = \frac{\sigma_{yc}}{\pi^2 nE}\left(\frac{L}{K}\right)^2 \qquad \text{for } L/K > 115 \qquad (8.1.3)$$

Where,

n = 1.0 for hinged end
n = 2.25 for fixed end
n = 1.6 for ends partly restrained, as in bearing
K = least radius of gyration, L = shaft length
σ_{yc} = yield stress in compression

Stress due to torsion

$$\tau_{xy} = \frac{16T}{\pi d_0^3 (1 - k^4)}$$

(8.1.4)

Where,

T : Torque on the shaft
τ_{xy} : Shear stress due to torsion

Combined Bending and Axial stress

Both bending and axial stresses are normal stresses, hence the net normal stress is given by,

$$\sigma_x = [\frac{32M}{\pi d_0^3 (1 - k^4)} \pm \frac{4\alpha F}{\pi d_0^2 (1 - k^2)}]$$

(8.1.5)

The net normal stress can be either positive or negative. Normally, shear stress due to torsion is only considered in a shaft and shear stress due to load on the shaft is neglected.

Maximum shear stress theory

Design of the shaft mostly uses maximum shear stress theory. It states that a machine member fails when the maximum shear stress at a point exceeds the maximum allowable shear stress for the shaft material. Therefore,

$$\tau_{max} = \tau_{allowable} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

(8.1.6)

Substituting the values of σ_x and τ_{xy} in the above equation, the final form is,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + T^2}$$

(8.1.7)

Therefore, the shaft diameter can be calculated in terms of external loads and material properties. However, the above equation is further standarised for steel shafting in terms of allowable design stress and load factors in ASME design code for shaft.

8.1.6 ASME design Code

The shafts are normally acted upon by gradual and sudden loads. Hence, the equation (8.1.7) is modified in ASME code by suitable load factors,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{C_{bm}M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + (C_t T)^2}$$

(8.1.8)

where, C_{bm} and C_t are the bending and torsion factors. The values of these factors are given below,

	C_{bm}	C_t
For stationary shaft:		
Load gradually applied	1.0	1.0
Load suddenly applied	1.5 - 2.0	1.5 - 2.0
For rotating shaft:		
Load gradually applied	1.5	1.0

Load suddenly applied (minor shock)	1.5 - 2.0	1.0 - 1.5
Load suddenly applied (heavy shock)	2.0 - 3.0	1.5 - 3.0

ASME code also suggests about the allowable design stress, $\tau_{allowable}$ to be considered for steel shafting,

ASME Code for commercial steel shafting

= 55 MPa for shaft without keyway

= 40 MPa for shaft with keyway

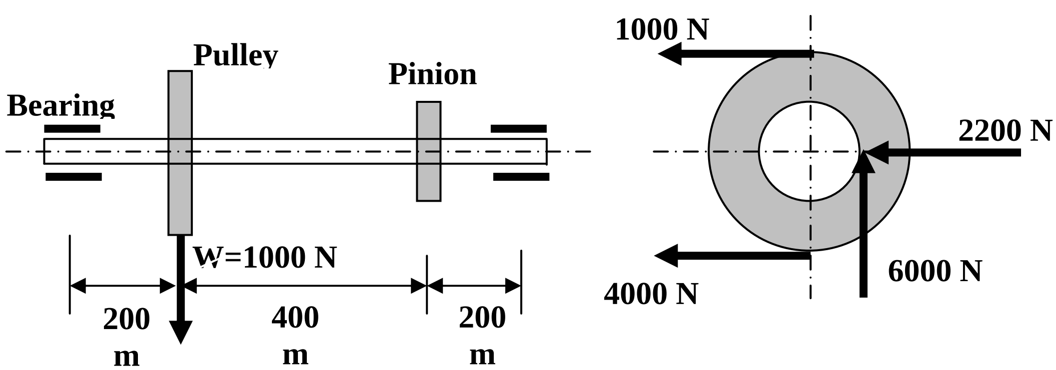
ASME Code for steel purchased under definite specifications

= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways.

The equations, (8.1.7) and (8.1.8) are commonly used to determine shaft diameter.

Sample problem

The problem is shown in the given figure. A pulley drive is transmitting power to a pinion, which in turn is transmitting power to some other machine element. Pulley and pinion diameters are 400mm and 200mm respectively. Shaft has to be designed for minor to heavy shock.

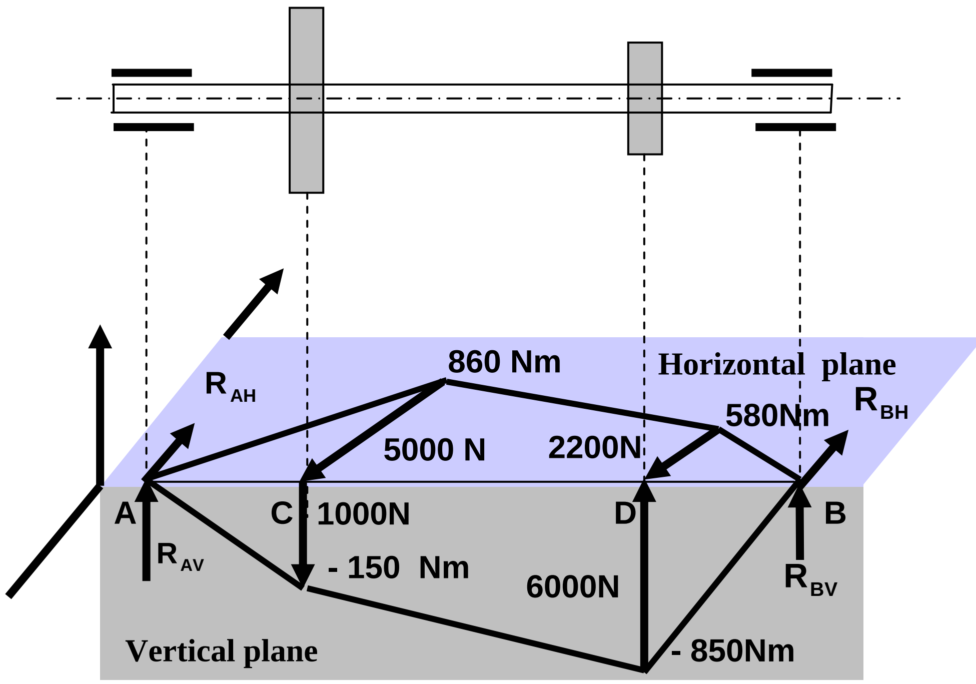


Solution

From the given figure, the magnitude of torque,

$$T = (4000 - 1000) \times 200 \text{ N mm} = 600 \times 10^3 \text{ Nmm}$$

It is observed that the load on the shaft is acting both in horizontal and vertical planes. The loading diagram, corresponding bearing reactions and bending moment diagram is given below.



Loading and Bending Moment Diagram

The bending moment at C:

For vertical plane, M_V: -150 Nm
For horizontal plane, M_H: 860 Nm
Resultant moment: 873 Nm

The bending moment at D:

For vertical plane, M_V: -850 Nm
For horizontal plane, M_H: 580 Nm
Resultant moment: 1029Nm

Therefore, section-D is critical and where bending moment and torsion is 1029 Nm and 600 Nm respectively.

ASME code for shaft design is suitable in this case as no other specifications are provided. In absence of any data for material property, the allowable shear for commercial steel shaft may be taken as 40 MPa, where keyway is present in the shaft.

For the given codition of shock, let us consider C_{bm} = 2.0 and C_t = 1.5.

From the ASME design code, we have,

$$d_o{}^3 = \frac{16\times10^3}{\tau_d\times\pi}\left(\sqrt{(C_{bm}\times1029)^2+(C_t\times600)^2}\right)$$
$$=\frac{16\times10^3}{40\times\pi}\left(\sqrt{(2.0\times1029)^2+(1.5\times600)^2}\right)$$
$$\therefore d_o = 65.88\text{ mm}\approx 66\text{mm}$$

From standard size available, the value of shaft diameter is also 66mm.

Questions and answers

Q1. What do you understand by shaft, axle and spindle?

A1. Shaft is a rotating member, in general, has a circular cross-section and is used to transmit power. Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque. Spindle is simply defined as a short shaft.

Q2. What are the common ferrous materials for a shaft?

A2. Common materials for shaft are, hot-rolled plain carbon steel, cold-drawn plain carbon/alloy composition and alloy steels.

Q3. How do the strength of a steel material for shafting is estimated in ASME design code for shaft?

A3. Material property for steel shaft for ASME code is as follows,
For commercial steel shafting
= 55 MPa for shaft without keyway
= 40 MPa for shaft with keyway
For steel purchased under definite specifications
= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways in the shaft.