STRUCTURAL MECHANICS

CLASSIFICATION OF MATERIAL

- ELASTIC MATERIALS
- PLASTIC MATERIALS
- DUCTILE MATERIALS
- BRITTLE MATERIALS

TEST FOR MECHANICAL PROPERTIES

- TENSILE TEST
- COMRESSIVE TEST
- IMPACT TEST
- FATIGUE TEST
- TORSION TEST

- **LOAD & ITS TYPES**

  External force acting on body is called load.

  TYPES

  Tensile

  Compressive

  Shearing

  Bending

  Twisting

- STRESS – Internal resistance per unit area of cross section offered by abody against applied force.

TYPES OF STRESS –

1. Direct stress

- Tensile & Compressive

2. Shear stress

STRAIN

- Ratio of change in dimension to original dimension is known as strain.
- Strain is dimensionless.

TYPES OF STRAIN

- Tensile stain
- Compressive strain
- Shearing strain
- Volumetric strain

HOOKES LAW

With in proportion limit stress directly proportional to strain.

ELASTIC CONSTANT

- Modulas of elasticity (E)
- Modulas of rigidity (G)
- Bulk modulas (K)
- Poissons ratio (μ)

RELATION BETWEEN ELASTIC CONSTANT

- $E=2G(1+\mu)$
- $E=3K(1-2\mu)$
- $E=9KG/3K+G$

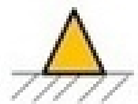
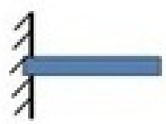
FACTOR OF SAFETY

Ratio of ultimate stess to working stress.

Shear Force &
Bending Moment

TYPES OF SUPPORT ON BEAMS

- **Free support–** When the beam rest freely on support.
- **Hinged support –** The beam supported on hinged support can rotate about hinge but can not move sideways.
- **Roller support–** The support in which beam free to move in horizontal direction.
- **Fixed support–** The support in which beam is fixed in position as well as in direction.

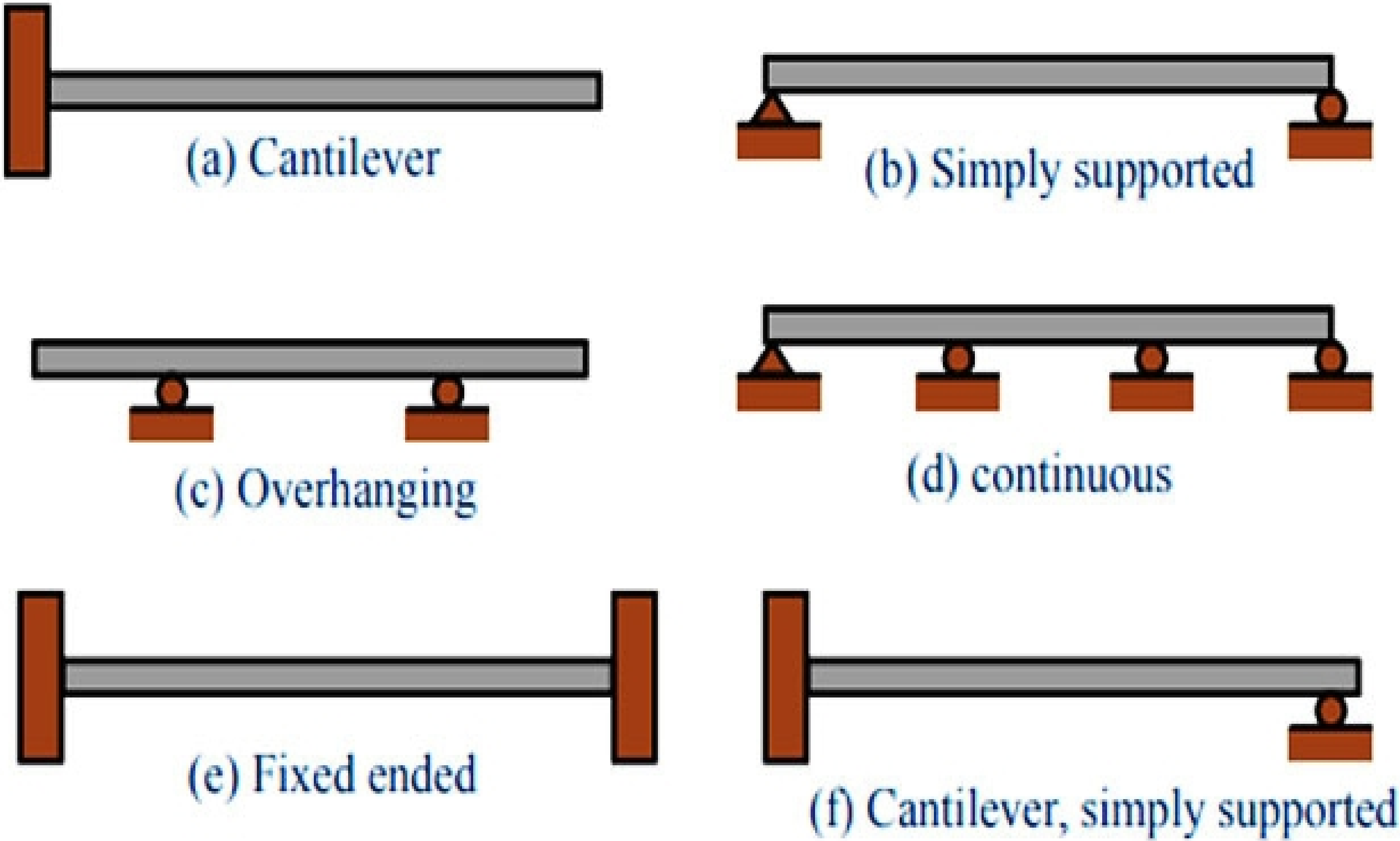
S.no	Types of Support	Representation by	Reaction Force	Resisting Load
1.	**Roller Support**		Vertical	Vertical loads
2.	**Pinned Support**		Horizontal and vertical	Vertical and horizontal loads
3.	**Fixed Support**		Horizontal, vertical and moments	All types of loads Horizontal, vertical and Moments
4.	**Simple Support**		Vertical	Vertical loads

CLASSIFICATION OF BEAMS

- **Cantilever beam** – A beam which is free at one end and fixed at one end.
- **Simply supported beam** – A beam in which both the ends freely resting on supports.
- **Overhanging beam** – A beam in which the supports are not situated at the ends.
- **Propped cantilever beam –** A beam in which one end is fixed and other end is

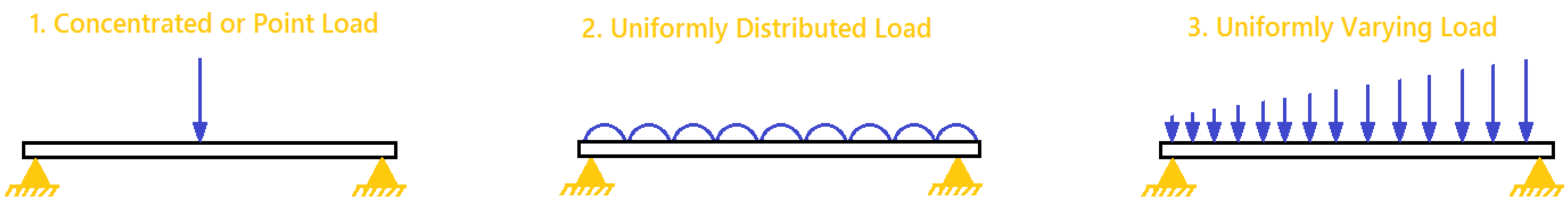
- **Fixed beam** – a beam which is rigidly fixed at ends.
- **Continuous beam** – a beam having more than two supports.

Types of Beams



TYPES OF LOAD

- **Point load** – it acts over a small area.
- **Distributed load** – it acts over a finite length.



- **Shear force** – The algebraic sum of all the vertical load acting on the beam on either side of section under consideration.
- **Shear force diagram** – It represents the variation of shear force along length of beam.

- **Bending moment** – The algebraic sum of moment of all vertical load about that section.
- **Bending moment diagram** – It represents the variation of bending moment along the length of beam.

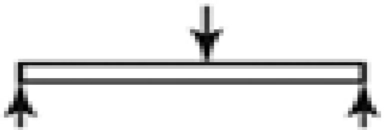
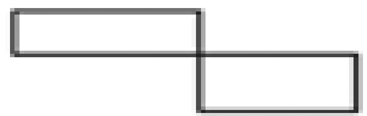
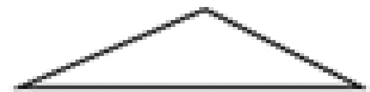
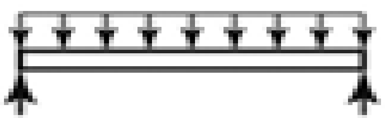
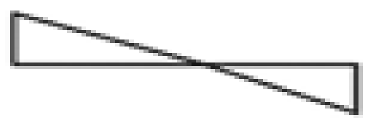
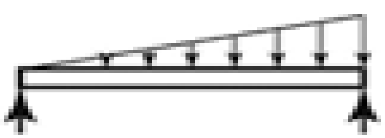
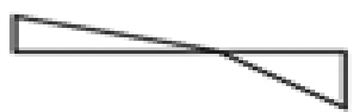
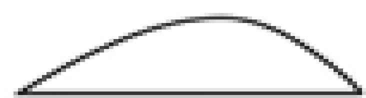
Load	Slope for shear force	Slope for bending Moment
P 	Constant 	Linear 
Uniformly distributed load 	Linear 	Parabolic 
Uniformly varying load 	Parabolic 	Cubic 

Figure-1 Slopes for various types of loads

Moment of inertia

Moment of inertia

- It is defined as the sum of second moment of area of individual section about an axis.

Second moment of area

- The product of the area and the square of distance of the centroid of the area from refrence axis is known as second moment of inertia.

Radius of gyration

- The distance from a given axis up to a point where the entire area is assumed to be considered.

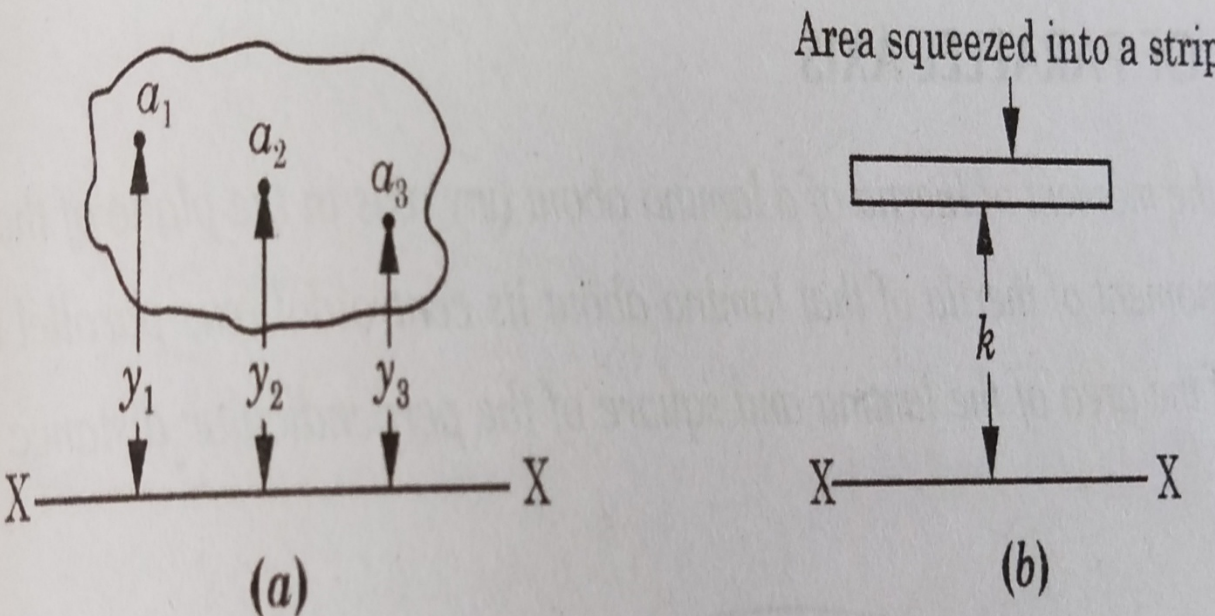


Fig. 4.1 : Moment of Inertia

As distance k is constant,

$$\therefore \qquad I_{XX} = k^2 \, \Sigma a = k^2 \, A$$

... given by equation (i) remains the s

Parallel axis theorem

- It states that the moment of inertia of any given figure about an axis parallel to the centroidal axis is taken as the sum of moment of inertia about the centroidal axis and the product of area and distance square between the two axis.

axes.

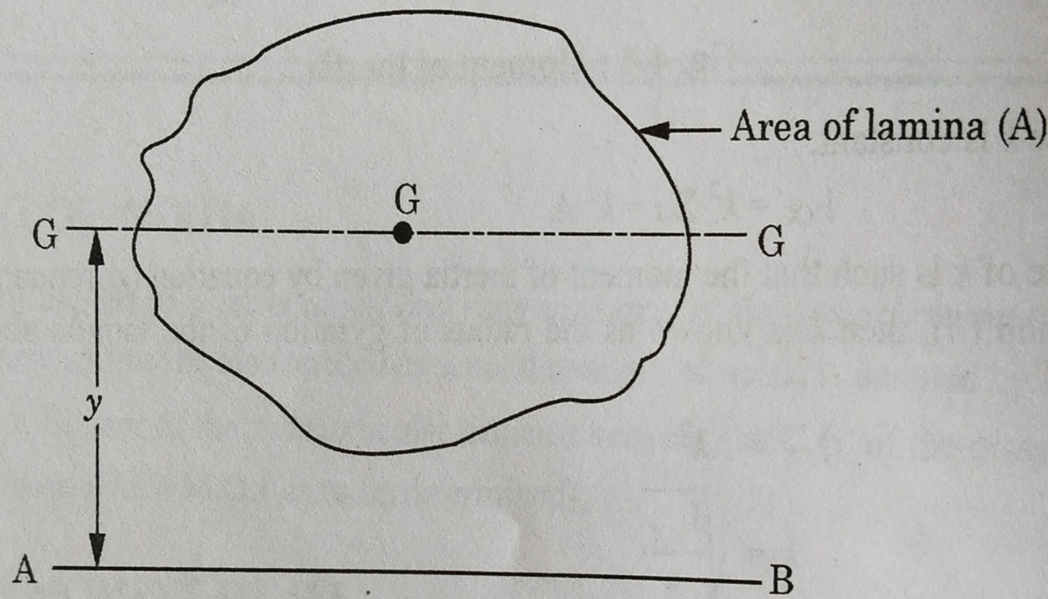


Fig. 4.3 : Theorem of Parallel Axis

Let us consider a lamina of area A as shown in fig. 4.3. Let G is the centroid of the lamina.

Now, $I_{AB} = I_{GG} + Ay^2$

where I_{AB} = Moment of inertia of lamina about axis AB,

I_{GG} = Moment of inertia of lamina about centroidal axis GG,

y = Distance between two axes.

Perpendicular axis theorem

- It states that if there is MOI in x & y axis than there is also MOI which is perpendicular to plane figure and both MOI (x and y axis).
- It is also called polar axis theorm.
- It is denoted by (J).

Let us consider a lamina of area A as shown in fig. 4.4.

Now, $I_{ZZ} = I_{XX} + I_{YY}$

where I_{ZZ} = Moment of inertia of the lamina about axis ZZ perpendicular to the lamina,

I_{XX}, I_{YY} = Moment of inertia of the lamina about axes XX and YY respectively in the plane of lamina.

The axis ZZ is generally known as polar axis and therefore, I_{ZZ}, the polar moment of inertia.

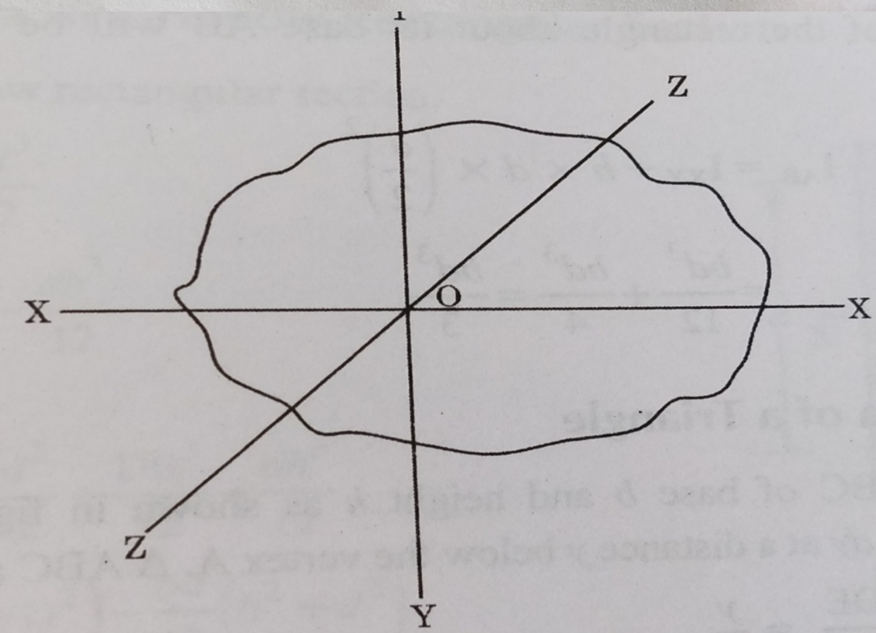
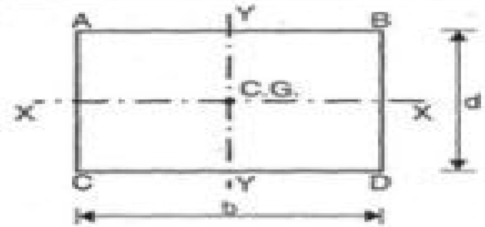
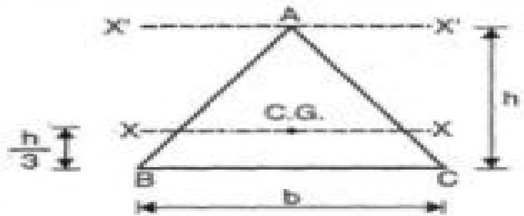
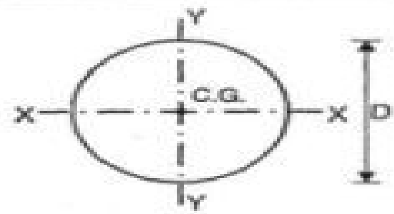
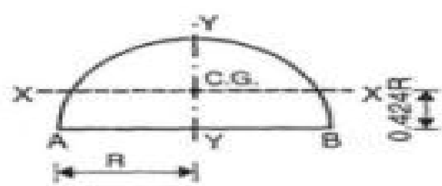
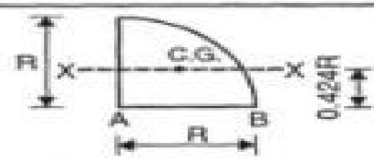


Fig. 4.4 : Theorem of Perpendicular Axis

Section modulus

- It is the ratio of moment of inertia of whole section about N.A to the distance from N.A to extreme fiber.

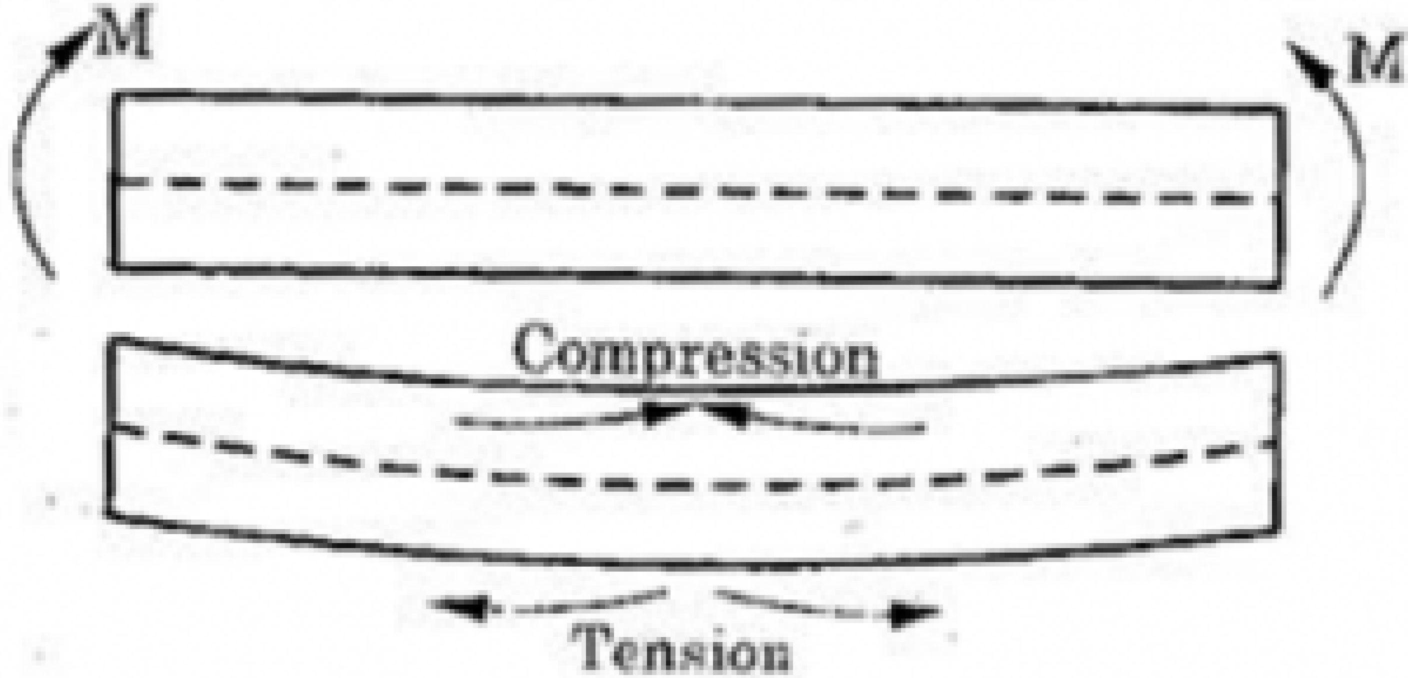
MOMENT OF INERTIA OF COMMON PLANE GEOMETRICAL FIGURES

Name of the Figure	Shape	Moment of Inertia (I)
Rectangle		$I_{XX} = \frac{bd^3}{12}$ $I_{YY} = \frac{db^3}{12}$ $I_{CD} = \frac{bd^3}{3}$
Triangle		$I_{X'X'} = \frac{bh^3}{4}$ $I_{XX} = \frac{bh^3}{36}$ $I_{BC} = \frac{bh^3}{12}$
Circle		$I_{XX} = I_{YY} = \frac{\pi D^4}{64}$
Semi-circle		$I_{XX} = 0.11\ R^4$ $I_{AB} = \frac{\pi R^4}{8}$ $I_{YY} = \frac{\pi R^4}{8}$
Quadrant		$I_{XX} = 0.0549\ R^4$ $I_{AB} = \frac{\pi R^4}{16}$

THEORY OF SIMPLE BENDING

BENDING STRESSES

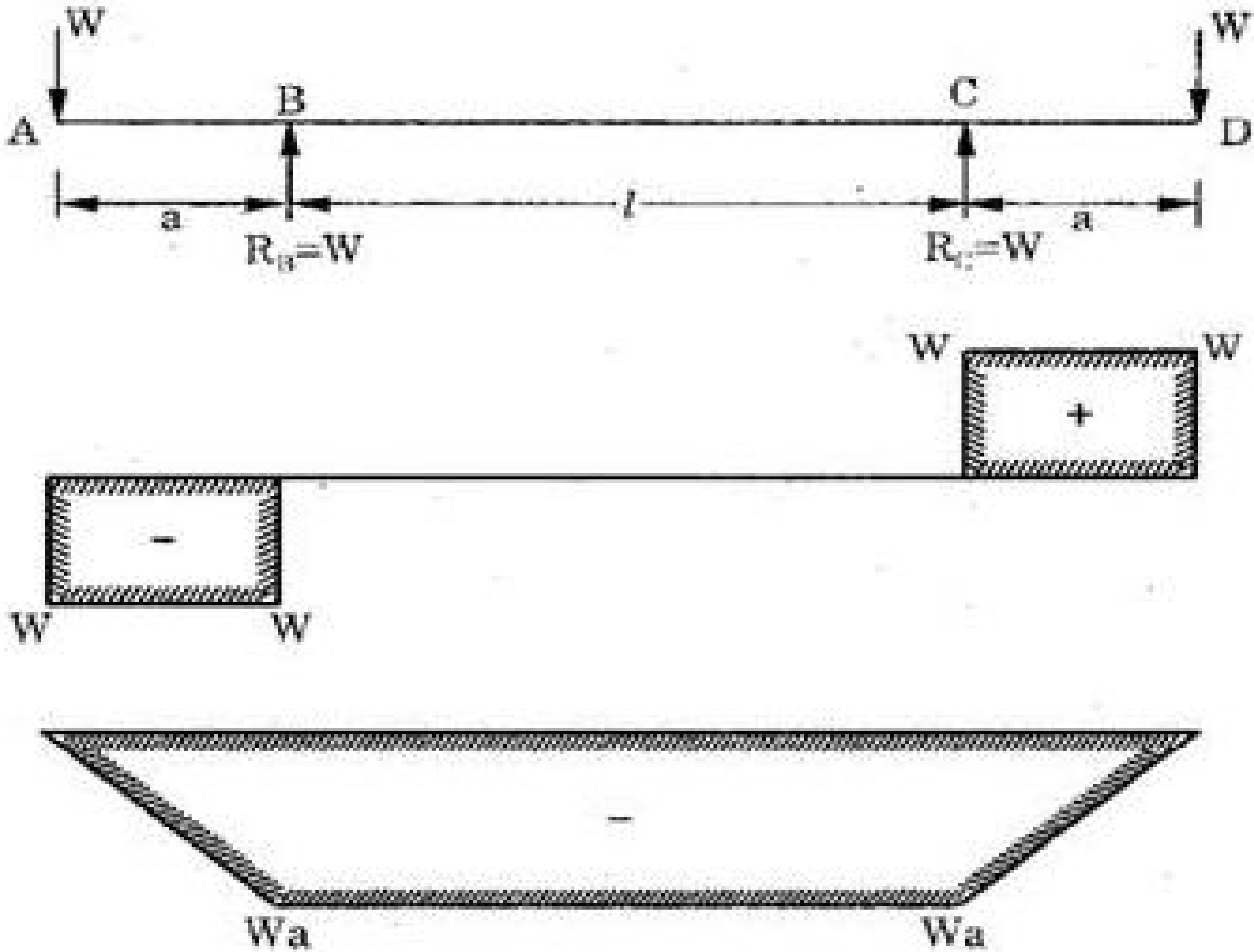
- When a horizontal beam is subjected to external load, shear force and bending moment are induced at all the sections of beam.
- Due to this shear force and bending moment, there will be a certain degree of deformation in the beam
- The beam will resist deformation.
- The resistance induced to resist bending is known as bending stress.
- The resistance induced to resist shearing is known as shearing stress.


M
M
Compression
Tension

Pure bending or simple bending

- A span is said to be pure bending if it is not subjected to shear force.

- Let us consider a beam ABCD with equal overhang and supported at B and C .
- Let a point load W be applied each end of beam.
- If we draw SFD & BMD for a given loading we find that there is no shear force between B and C but the bending moment is constant between B & C.
- This means that between B and C beam is subjected to a constant bending moment only and no shear force therefore it is a case of pure bending.



W
W
A
B
C
D
a
l
a
$R_B = W$
$R_C = W$
W
W
+
-
W
W
-
Wa
Wa

Stresses in Beams – Bending and Shear

- Assumptions in simple (pure) bending theory:
  - Material of beam is homogenous and isotropic (same composition & constant E in all directions).
  - Young's modulus is constant in compression and tension.
  - Transverse section which are plane before bending remain plain after bending (Eliminate effects of strains in other direction).
  - Beam is initially straight and all longitudinal filaments bend in circular arcs.
  - Radius of curvature is large compared with dimension of cross sections.
  - Each layer of the beam is free to expand or contract.

Combined direct and bending stress

Introduction

- When a body is subjected to axial load ,direct stress is induced.
- When a body is subjected to bending moment, a bending stress is induced in it.
- To find the resultant stress these two stresses must be algebraically added.

Eccentric load

- The load whose line of action does not coincide with the axis of column or strut.

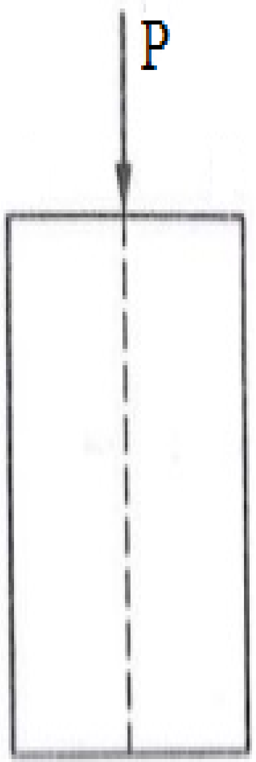


Fig : 1

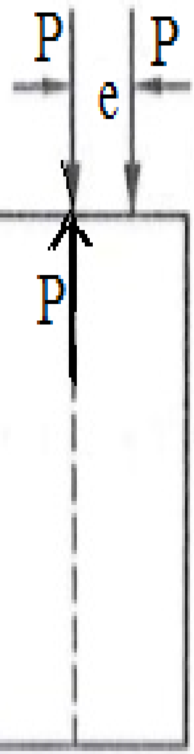


Fig: 2

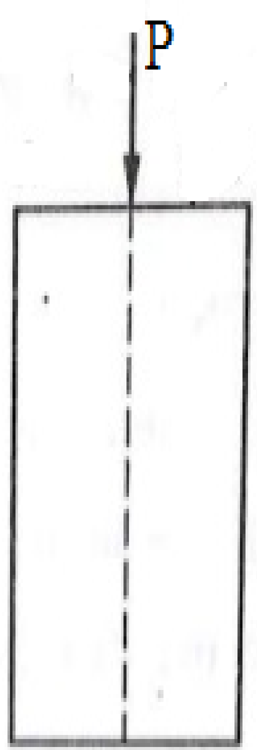


Fig:3

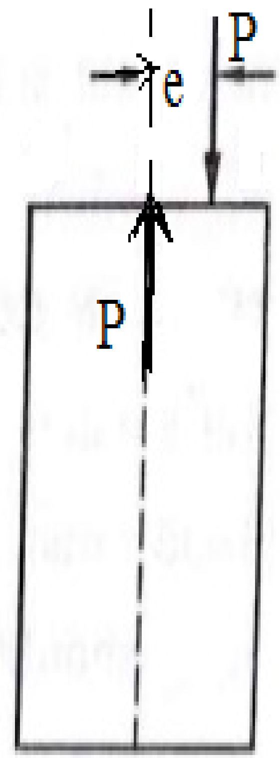
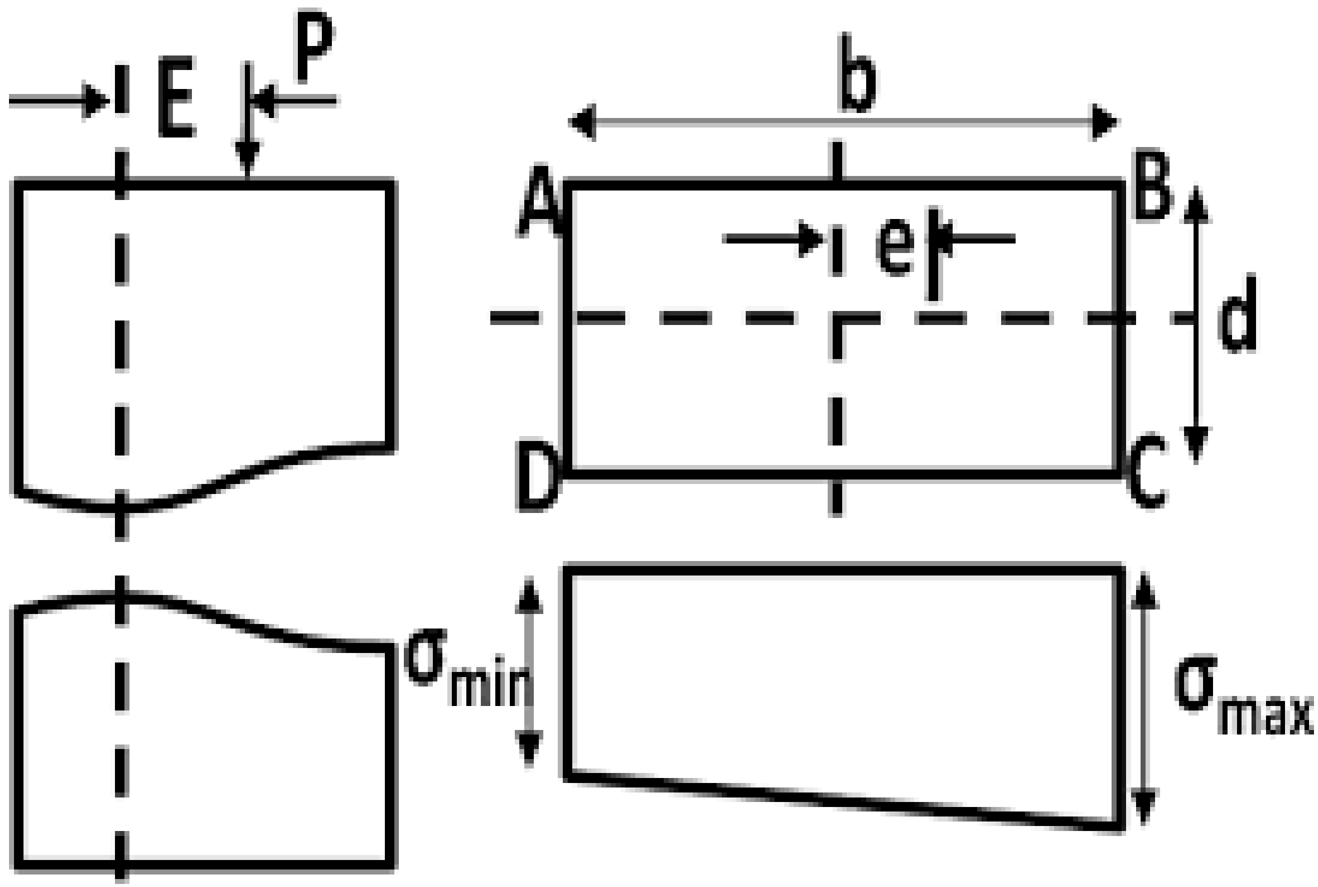


Fig:4

- Let us introduce two equal and opposite forces P along the axis of column as shown in fig. 2.
- One of those forces acting along the axis of column causes a direct stress as shown in fig.3.
- Other two forces form a couple of moment equal to P X e which causes bending stress.
- We can say that if a column is subjected to eccentric load, direct as well as bending stress will induced in column.

Symmetrical column with eccentric loading about one axis



- Let us consider a column ABCD subjected to an eccentric load P as shown in fig.
- Let e=eccentricity of load, b=width of column, d=thickness of column.

Direct stress $\sigma = P/A$

Bending stress $\sigma_b = M/Z$

- Total stress will be maximum at top & minimum on bottom.

$$\sigma_{max.} = \sigma + \sigma_b$$

$$\sigma_{min.} = \sigma - \sigma_b$$

Limit of eccentricity

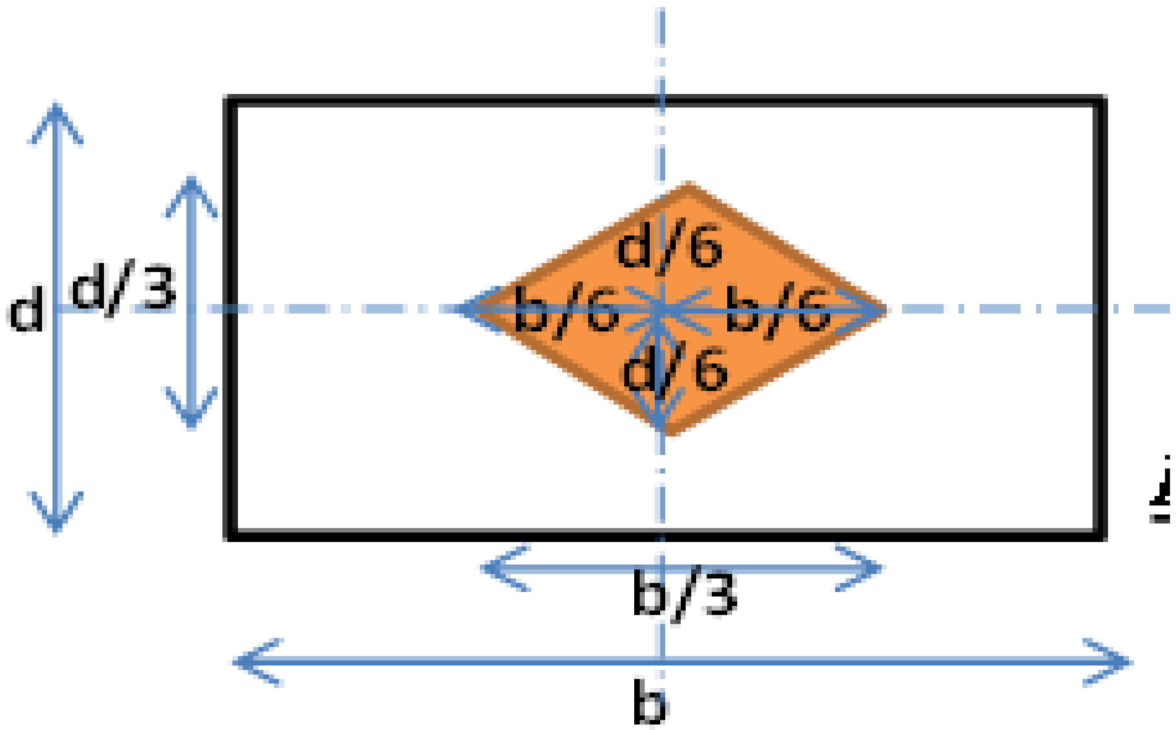
- If bending stress is more than direct stress then there will be tensile stress on one side.
- If bending stress is equal to direct stress then there will be zero stress on one side.
- If bending stress is less than direct stress then resultant stress will be compressive.
- It should be noted that no tensile sress

$$\sigma_b \leq \sigma$$

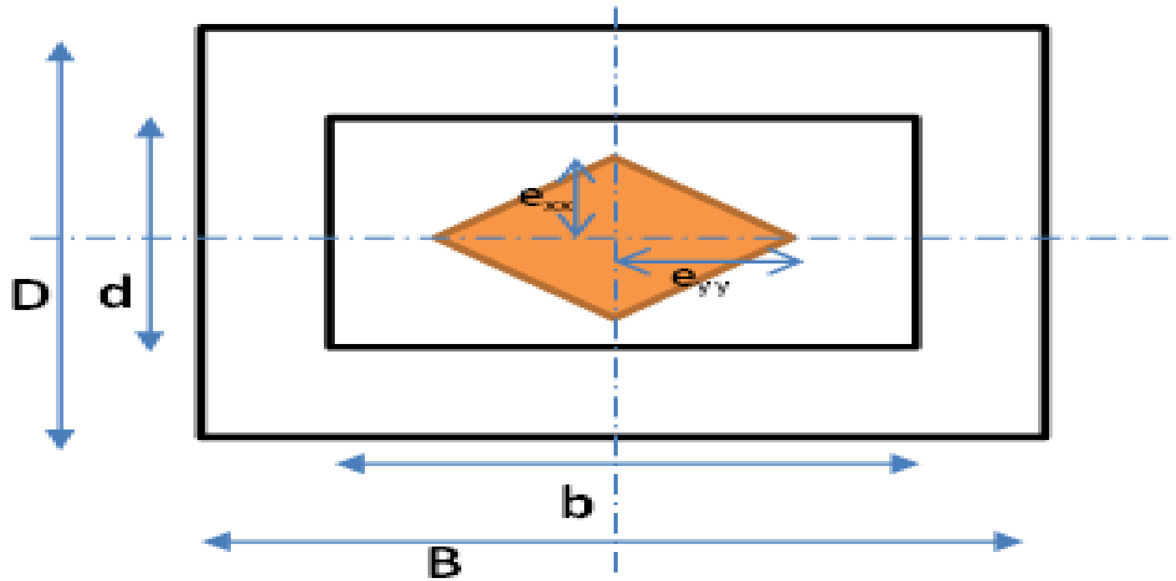
$$e \leq Z/A$$

- For rectangular section $e \leq d/6$
- For circular section $e \leq d/8$

Limit of eccentricity of rectangular section

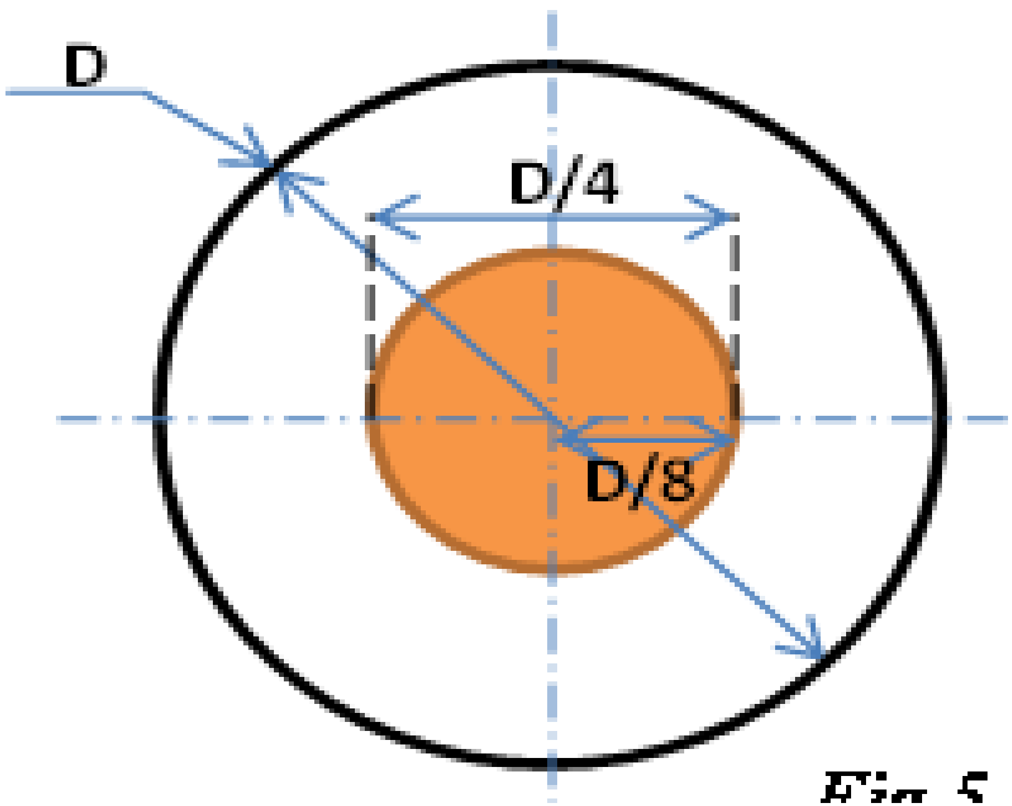


Limit of eccentricity of hollow rectangular section



$$e_{xx} = \frac{BD^3 - bd^3}{(BD - bd)\cdot 6D};\quad e_{yy} = \frac{DB^3 - db^3}{(BD - bd)\cdot 6B}$$

Limit of eccentricity of circular section



Limit of eccentricity of hollow circular section

$$e = \frac{(D^2 + d^2)}{8D}$$

$dia. of\ core\ section$

$$= \frac{(D^2 + d^2)}{4D}$$

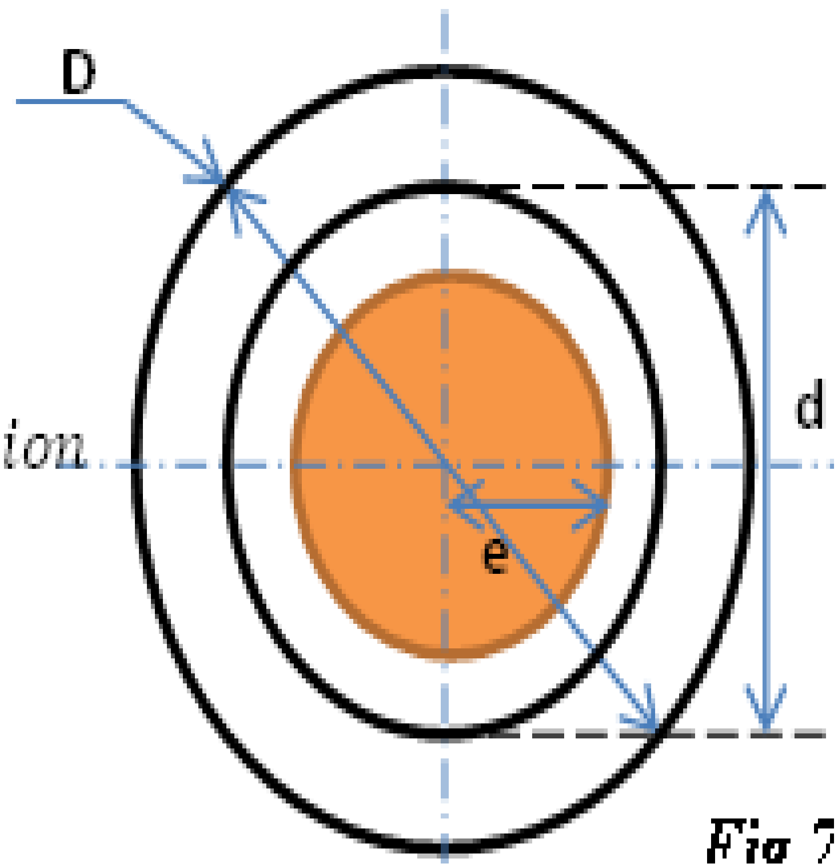


Fig 7

Dam

- It is a structure constructed to store a large quantity of water which is used for irrigation and power generation purpose.

<u>Types of dams</u>

- Rectangular dam
- Trapezoidal dam

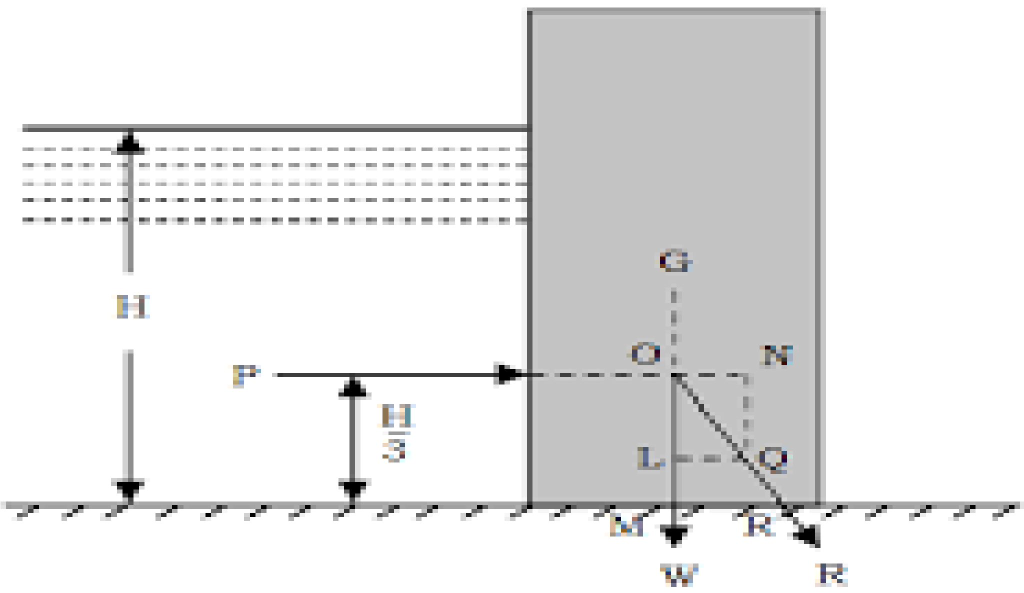


Fig : Rectangular Dam

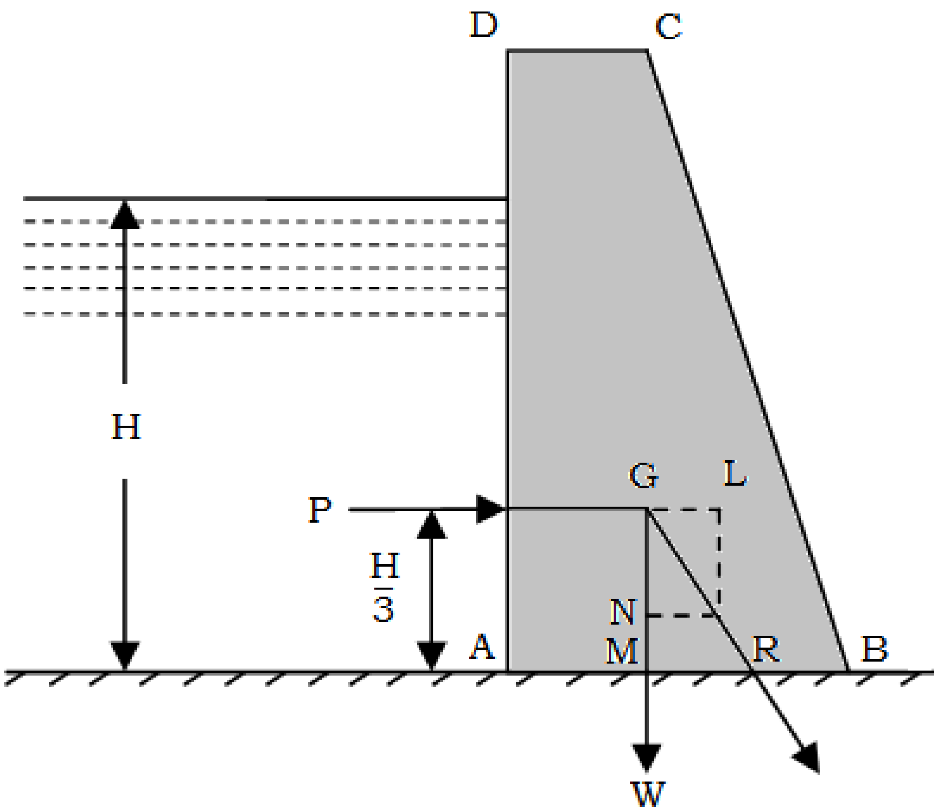


Fig : Trapezoidal Dam

Conditions for the stability of dam

- To avoid tension in the masonry at the base of dam
- To safeguard the dam from overturning
- To prevent sliding of the dam
- To prevent crushing of masonry at the base of dam.

Retaining wall

- It is generally constructed to retain earth on one of its faces in hilly areas.

Earth pressure on a retaining wall

- Active earth pressure – the pressure exerted by the retained material called backfill on the retaining wall
- Passive earth pressure – sometimes the retraining wall moves laterally against the retained earth which gets compressed . Due to this compressed earth is subjected to a pressure called passive earth pressure.

Conditions for the stability of a retaining wall

- To avoid tension in the masonry at the base of the wall
- To safeguard the wall from overturning
- To prevent sliding of the wall
- To prevent crushing of masonry at the base of the wall

COLUMNS

- Strut – a bar or a member of a structure in any position other than vertical subjected to an axial compressive load
- Column – a bar or a member of structure inclined at 90° to the horizontal and carrying an axial compressive load
- Slenderness ratio – the ratio between length of a column and the minimum radius of gyration
- Buckling/crippling/critical load – the minimum limiting load at which the column tends to buckle

- Safe load – the load to which a column is subjected and which is below the buckling load
- Crushing load – the maximum axial compressive load which a column can take without failure by crushing

SLOPE AND DEFLECTION

INTRODUCTION

When a beam is subjected to a load , it deflects from its original position.

Deflection depends upon cross section of beam and applied load.

It should be noted that beam should not deflect more than permissible limit.

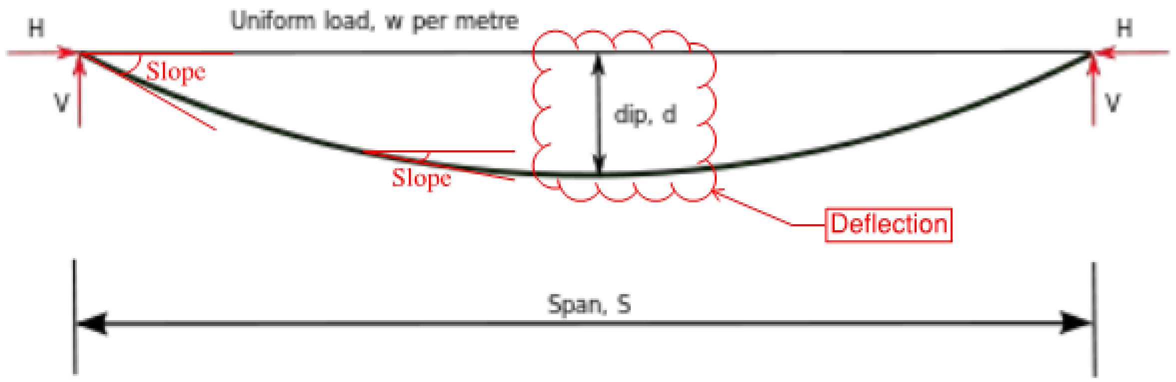
DEFLECTION

The vertical displacement of the neutral axis of the beam at the section due to bending.

Neutral axis of beam bend into curve. This curve is known as elastic curve of loaded beam.

SLOPE

The angle made by the tangent drawn at section of the deflected beam with the horizontal.



NECESSITY OF DETERMINING DEFLECTION

While designing a beam its safety is checked in bending, shearing and deflection.

For safety this deflection produced in the beam should be well with in the permissible limits.

If deflection exceeds permissible limit then surface finishing material will be damaged.

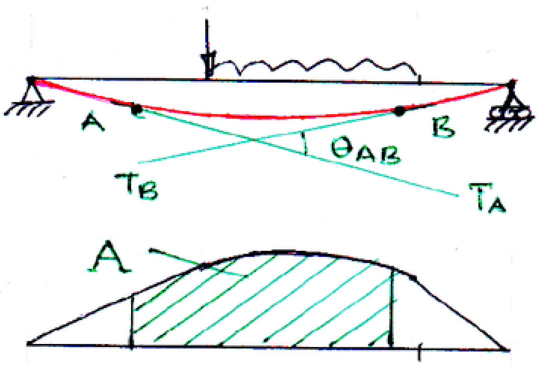
MOMENT AREA METHOD

The method for determining slope and deflection by considering the area of the bending moment diagram is known as moment area method.

Moment Area Theorems:

Theorem 1:

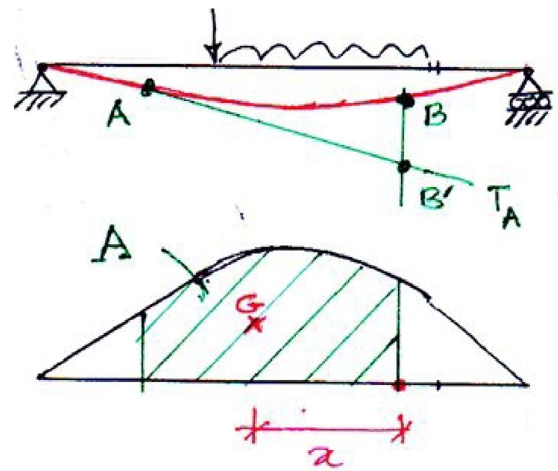
When a beam is subjected to external loading, it under goes deformation. Then the intersection angle between tangents drawn at any two points on the elastic curve is given by the area of bending moment diagram divided by its flexural rigidity.

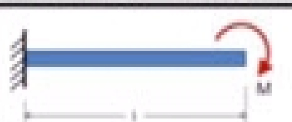
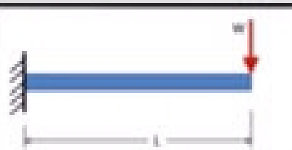
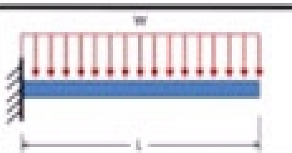
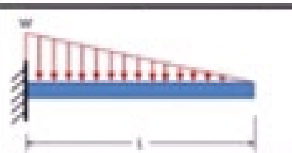
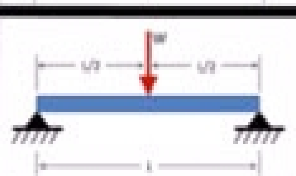
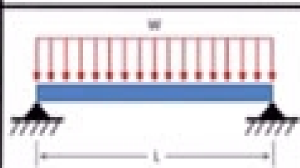


Moment Area Theorems:

Theorem 2:

The vertical distance between any point on the elastic curve and intersection of a vertical line through that point and tangent drawn at some other point on the elastic curve is given by the moment of area of bending moment diagram between two points taken about first point divided by flexural rigidity.



SR. NO.	TYPE OF BEAM	MAX. BM	SLOPE	DEFLECTON
1		M	$\theta = \frac{ML}{EI} = \frac{ML}{EI}$	$\delta = \theta \times \frac{L}{2} = \frac{ML^2}{2EI}$
2		WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\delta = \theta \times \frac{2L}{3} = \frac{WL^3}{3EI}$
3		$\frac{WL^2}{2}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{6EI}$	$\delta = \theta \times \frac{3L}{4} = \frac{WL^4}{8EI}$
4		$\frac{WL^2}{6}$	$\theta = \frac{ML}{4EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{4L}{5} = \frac{WL^4}{30EI}$
5		$\frac{WL}{4}$	$\theta = \frac{ML}{4EI} = \frac{WL^2}{16EI}$	$\delta = \theta \times \frac{L}{3} = \frac{WL^3}{48EI}$
6		$\frac{WL^2}{8}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{5L}{16} = \frac{5WL^4}{384EI}$